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Forward jets and large rapidity gaps

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Hadronic jets are extremely abundant at the LHC, and testing QCD in various corners of phase-space is important to understand backgrounds and some specific signatures of new physics. In this article, various measurements aiming at probing QCD in configurations where the theory modeling become challenging are presented. Azimuthal angle de-correlations are sensitive to hard as well as soft QCD emission, and in most of the events jets are produced in a back-to-back configuration. Events where jets have a large rapidity separation are also rare, and those without additional radiation between the jets are exponentially suppressed. The modeling of radiation between very forward and backward jets is complicated, and may require theoretical tools different with respect to those normally used for central, high-pt events. Observables can be created that are sensitive to all these effects, like the study of azimuthal angle de-correlations between events where the two leading jets have large rapidity separations. The two general-purpose detectors of the LHC have measured these observables, and for some of them interesting deviations with respect to the most commonly used theoretical models are observed.

1. Introduction

Production of hadronic jet pairs is the most common high-momentum transfer process at the LHC, and has been widely studied, first with early 2010 data,[1,2] then with much larger datasets (e.g.[3,4]). Dijet events are used to search for new physics, in particular resonances[5,6] decaying into quark or gluon final states. Most of the standard measurements of dijet production are limited to jets of high transverse momentum in central pseudorapidities ($|\eta| < 3$). In this kinematic configuration, the momentum fractions of the incoming partons are relatively large and of similar order of magnitude, and in regions where strong constraints exist to the Parton Distribution Functions (PDF's) from deep-inelastic scattering data. Also, for this high-x kinematical configuration, it is expected that the standard DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)[7] evolution equations provide a good approximation of the underlying physics, and in fact no sizeable deviations from the expected behaviour are observed. More interesting tests of QCD can be performed in specific corners of phase-space, where the modeling of the underlying physics may be less obvious. For instance, events with large azimuthal de-correlations are

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sensitive to higher-order emission of hard gluons. Events with at least one jet in the forward direction can arise from an imbalance of the momentum fraction of the two partons, and therefore probe less-constraint regions of the PDFs. Events where the two leading jets in the event have a large rapidity separation may be better described by theoretical models involving multiple scales and large logarithmic constributions. These configurations may require the use of different evolution equations than DGLAP, like the approach from Balitski-Fadin-Kuraev-Lipatov (BFKL),[8] from Ciafaloni-Catani-Fiorani-Marchesini (CCFM)[9] or inspired by gluon saturation.[10]

Evidence for deviations from the DGLAP description in systems of forward jets has been searched for in various experiments and colliders before the LHC. D0 measured forward-backward jets with rapidity separations up to 6,[11] while both the ZEUS[12] and H1[13] collaborations studied the systems with forward jets in the final state. Since these measurements did not give any compelling evidence for a breakdown of the validity of the DGLAP approach, it makes sense to pursue the study at the LHC, where the instrumentation of the detectors in the very forward region allows probing even more extreme rapidity separations.

In addition to the exchange of gluons varying QCD color, dijet events could also be produced by the exchange of color-neutral gluon ladders. The probability of these kinds of processes is roughly independent on the rapidity separation between the dijets, while the more common color-singlet exchange has an exponentially decreasing dependence on the rapidity separation. The consequence is that events with large rapidity separation are proportionally more likely to be produced by color singlets, a configuration where no additional radiation between the jets is emitted. The experimental study of events with jet veto, where only the events without hard emission in the rapidity region between the two leading dijets are selected, can enhance the occurrence of interesting regions of phase space. Combination between some of the techniques described above (like for instance the study of azimuthal de-correlations for events with and without jet veto) can probe even more specific regions of phase-space, highlighting the potentially different ability to describe the data by the various theoretical approaches.

In the following sections, several measurements are presented where QCD is probed in specific corners of phase-space, with specific emphasis on measurements in the forward region of the detectors.

2. Early azimuthal decorrelation measurements

The first measurements to test QCD in difficult regions of phase-space of the dijet system have been dijet azimuthal decorrelations. At Born level, the outgoing partons in a $2 \rightarrow 2$ interaction are produced exactly back-to-back in the azimuthal plane, and with equal transverse momentum. Hadronisation effects leading from partons to jets little change this picture, so in the absence of extra radiation the angle between the two jets is supposed to be very close to π, and indeed this is the case for the majority

of the events. Soft gluon radiation will contribute to small deviations with respect to the back-to-back configuration, but it is only hard gluon radiation, resulting in multijet final states, that can lead to significant deviations from the back-to-back configuration. The measurement of azimuthal de-correlation is therefore a test of QCD in various regimes, as well as a probe for various Initial State Radiation (ISR) models, since hard ISR can boost the dijet system in the transverse plane. Figure 1 shows the distribution of the azimuthal angle as measured by ATLAS using the data collected in 2010 (left),[14] unfolded to particle level and compared to Next-to-Leading Order (NLO) QCD as computed by NLOJet++.[15] A different approach is the one shown in the right plot of that figure, where the ratio is shown of the aximuthal angle distribution from data collected by CMS[16] to the predictions from D6T PYTHIA6[24] tunes. In this case, the parameter responsible for ISR [PARP(67)] has been varied over a wide range of values (including the default of 2.5). It can be observed that the default value gives a very good description of the data, while for extreme values like 1 the discrepancy with data is large. Varying this parameter by ± 0.5 gives variations at the 30% level, showing that the azimuthal decorrelation is sensitive to ISR, and can be used to tune this parameter.

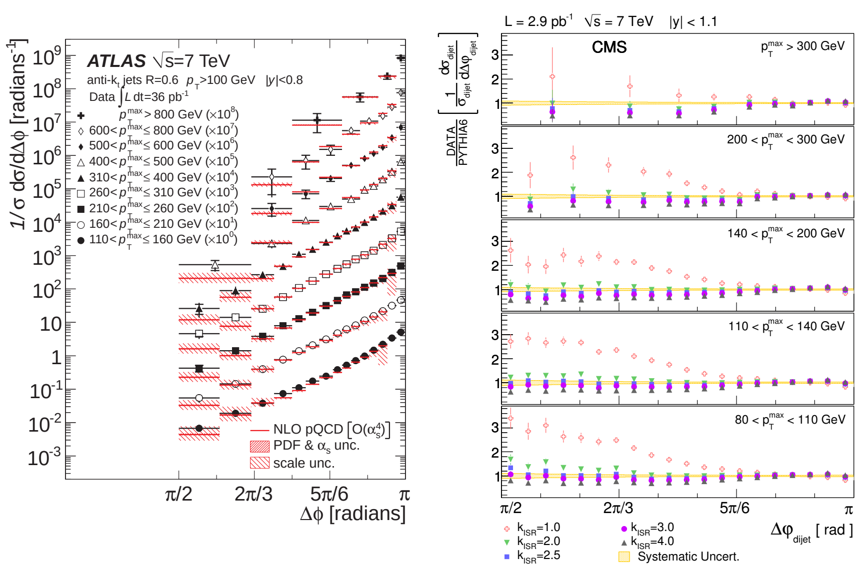

Fig. 1. Azimuthal angle distribution in various ranges of the transverse momentum of the leading jet. Left: data from ATLAS are compared to NLO QCD predictions; right: the ratio between data and the D6T PYTHIA6 tune, where the parameter governing ISR has been varied.

3. Cross section measurements in the forward region

The early measurements of azimuthal decorrelations were performed on jets in the central region, for reasons of trigger efficiency and uniformity of detector response. Measuring the cross section for inclusive jets and dijets in the forward region probes a region of the proton PDFs that is less constraint by deep-inelastic scattering data, and also where perturbative QCD makes less solid predictions. It was remarked, for instance, that for dijets with similar transverse momenta, the cross section has a strong dependence on the choice of renormalisation and factorisation scales, and that actually the usual choice of setting these values to the transverse momentum of the leading jet can lead to negative cross sections in some corners of the phase-space, such as for large rapidity separations. In the ATLAS publication[17] the theory prediction had a choice of scales that depends exponentially on y^*, half the absolute value of the dijet rapidity separation:

$$\mu = p_T \exp(0.3y^*)$$

CMS measured the inclusive cross section for forward jets, and for dijets where one of them is in the forward region, in Ref.[18] Unfolded data are compared to a series of theoretical models, including NLO Monte Carlo generators and BFKL-inspired ones, in the rapidity range $3.2 < |y| < 4.7$. The ratio of many theoretical models and data is shown in the left side of Fig. 2. Good agreement with all models is present, within an uncertainty of 20%, the same order of magnitude as the theoretical differences.

ATLAS included the measurements of the forward region in the inclusive and dijet cross section paper.[17] The right side of Fig. 2 shows the ratio of the ATLAS dijet cross section measurement with respect to theory predictions from NLOJET++, compared to the ratio of other theoretical models to the same denominator. The cross section is measured as a function of the dijet invariant mass, in various bins of the half rapidity difference y^*, of which the figure shows only the largest one. Here the differences between the various models, as well as the systematic uncertainties for some points, can be up to factors of 2 or 3, being the largest at low transverse momenta and large rapidity separations.

4. Dijets with large rapidity separations

The kinematic regime where two jets are separated by a large rapidity separation is where the standard modeling of QCD is expected to encounter more difficulties, and the approximations made in the DGLAP evolution to break down. Alternative approaches to these evolution equation have been developed, both as effective theories[19] and as Monte Carlo generators inspired by CCFM evolution like CASCADE[20] or BFKL like HEJ.[21] The regime in which these effects become relevant is unclear, so most of the experimental studies are performed as a function of the rapidity separation between the two jets.

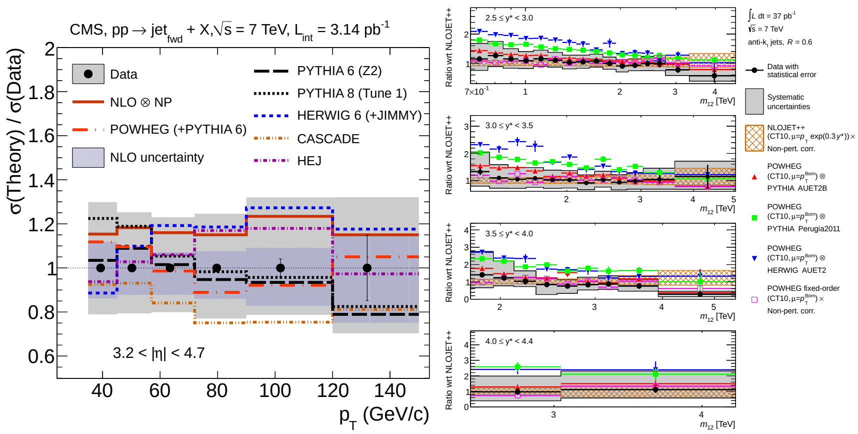

Fig. 2. The ratio between measured dijet cross section in the forward region for CMS (left) and ATLAS (right). Unfolded results from data are compared to analytical NLO QCD calculations, as well as LO and NLO Monte Carlo predictions.

ATLAS performed an explicit measurement of the number of additional jets in a dijet system, in the two cases when the boundary jets are defined as the two leading jets of the events, and when they are the most forward and backward of the event above a transverse momentum of 20 GeV (an approach inspired by the studies of Mueller and Navelet[22]).[23] An additional requirement is that the average transverse momentum of the two boundary jets should be larger than 50 GeV, to be in the fully-efficient region of the jet trigger. The measurements presented in Fig. 3 are performed as a function of the dijet rapidity separation, in bins of average transverse momentum of the two boundary jets. The left plot shows the case when the boundary jets follow the Mueller-Navelet definition (the most forward and backward above a threshold). The theory comparison is made with the NLO+PS generator POWHEG,[25] interfaced with both PYTHIA and HERWIG[26] for the showering, and with HEJ interfaced with ARIADNE. Also in this case, the DGLAP-based generators, especially POWHEG interfaced with PYTHIA, perform very well, giving an accurate description of the measured quantity over the whole phase-space. HEJ tends to generally underestimate the radiation in the rapidity interval, especially for the selection based on the leading jets in the event, and at large values of the average transverse momentum and rapidity separation. The right plot, from a successive publication,[27] shows the same quantity in an extended range of $\Delta\eta$, in the case when the boundary jets are the two leading jets of the event. The additional theory band when the parton-level predictions from HEJ are coupled to a parton shower from ARIADNE[28] is present, showing a much better agreement with data, even if the POWHEG+PYTHIA prediction is not significantly worse. Also for this measurement, no advantage is seen in the use of specific BFKL-based models.

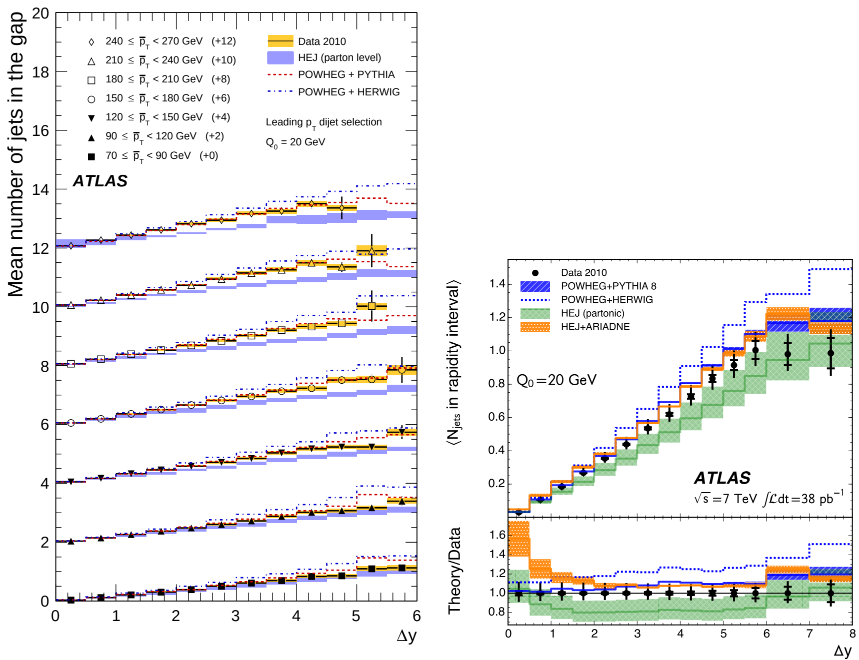

Fig. 3. Mean number of jets in a rapidity gap, as a function of the rapidity separation between the two leading jets in the event (left), and the separation between the most forward and most backward jet (right plot).

5. Study of jet veto in dijet events

CMS measured[29] the ratios between inclusive and exclusive dijet cross-sections $R^{\rm incl} = \sigma^{\rm incl}/\sigma^{\rm excl}$ and the ratio of the Mueller-Navelet jets to the exclusive ones $R^{\rm MN} = \sigma^{\rm MN}/\sigma^{\rm excl}$ as a function of the dijet rapidity separation. Events were considered if at least two jets with $p_T > 35$ GeV and absolute rapidity $|y| < 4.7$ were present. Inclusive events have at least two jets passing these criteria; exclusive events have exactly two jets of this kind. In the inclusive case, each pairwise combination of jets is considered to calculate the rapidity separation, so by construction the first ratio is always larger than one. Mueller-Navelet jets are the most forward and most backward jet of the event; so for the second ratio only one combination is taken, and by construction $R^{MN} \leq R^{incl}$; however, also R^{MN} is larger or equal than one, since, even if only one combination is taken, the number of events with jets according to the inclusive definition is larger than that of those with exclusive jets. At extreme rapidity separations, the two ratios tend to converge, since it is very rare to have more than one jet combination with very large $|\Delta y|$. The results of this measurement are shown in Fig. 4 for the two ratios R^{incl} and R^{MN}, respectively in the left and right plots. Data is compared to both DGLAP-inspired models like PYTHIA6, PYTHIA8 and HERWIG++ and to the generators CASCADE and

HEJ 1.3.2, whose parton-level jets coupled to the parton-shower model from ARIADNE 4.12. The increase of the two ratios with rapidity separation follows the expected opening of additional phase-space for additional parton radiation; as expected the two quantities become similar at large $|\Delta y|$. The comparison with theory shows that DGLAP-inspired generators (especially the two versions of PYTHIA) do a very good job at describing data even in the region of large rapidity separations, where they were supposed to perform badly; on the other hand, the two BFKL-inspired codes show large discrepancies with data, for both ratios.

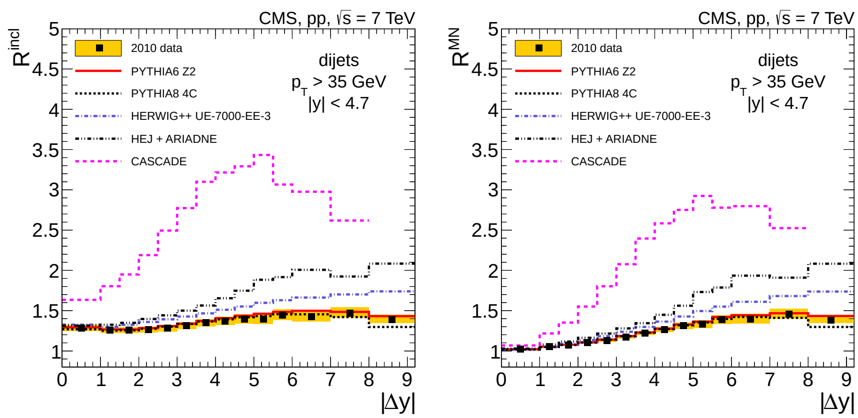

Fig. 4. Left: ratio of the inclusive to exclusive dijet cross section R^{incl} as a function of the rapidity separation; right: ratio of the Muller- Navelet to exclusive jet cross sections. Data with systematic error band (the statistical errors are smaller than the size of the symbol) is compared to several theoretical models. In addition to the DGLAP-based generators like PYTHIA6, PYTHIA8 and HERWIG++, also BFKL-inspired ones like CASCADE and HEJ+ARIADNE are shown.

ATLAS measured the "gap fraction", defined as the ratio between dijet events without a third one in the rapidity interval between the two main boundary jets and the total number of dijet events. As for the result on the number of jets in the gap, two definitions of boundary jets were used: the two leading jets in the event, and the most forward and backward above a given transverse momentum threshold. The ratio taken using the second definition is equivalent to the inverse of R^{MN} from the CMS paper. For these two definitions of boundary jets, ATLAS measures the gap fraction as a function of the rapidity difference for various bins of average transverse momentum. Figure 5 shows the gap fraction where the boundary jets are defined as the leading jets of the event; on the left side, data is compared to leading-order codes like PYTHIA, HERWIG and SHERPA, while on the right side it is compared to the NLO code POWHEG (coupled to parton shower by both PYTHIA and HERWIG), and to the resummed approach of HEJ. We see that among LO generators, PYTHIA gives a reasonable description of the data, while HERWIG and in particular SHERPA show increasing deviations at the largest rapidity separations. The combination of POWHEG and PYTHIA again gives the

best description of data, with only small deviations for large rapidity separations, at high or low values of the average boundary jet values; on the other hand, HEJ overestimates the gap fraction in large regions of the phase space.

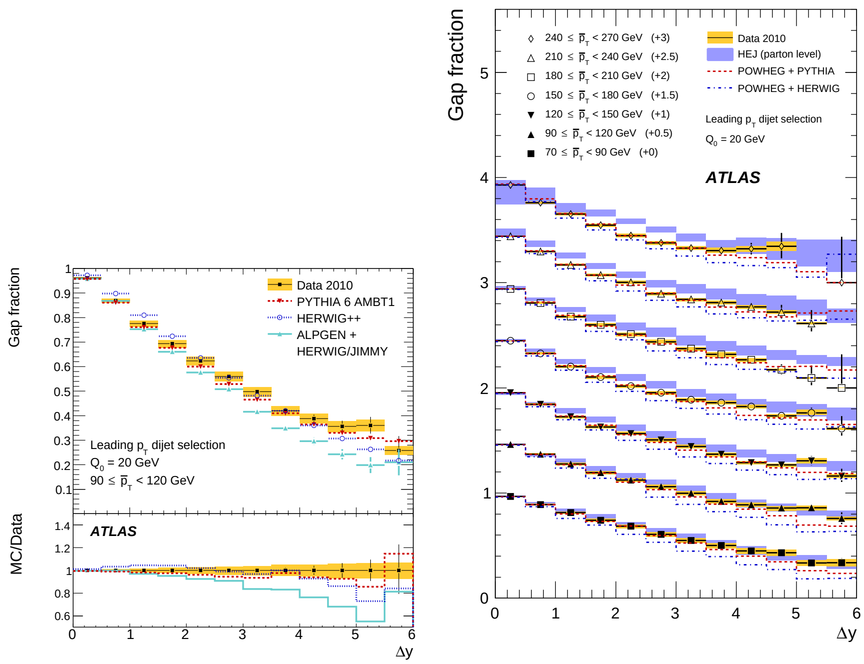

Fig. 5. Gap fraction as a function of the rapidity separation between the two leading jets in the event. Left: comparison of data to LO generators. Right: comparison with NLO and resummed generators.

Figure 6 shows instead the results obtained with the Mueller-Navelet approach, when the two boundary jets are defined as the most forward and most backward of the event. The left plot shows the gap fraction as a function of the dijet rapidity separation, and is therefore directly comparable with the right side of Fig. 5, but with a different boundary jet definition; the right side shows instead the gap fraction as a function of the thresholds on the transverse momentum of the veto jet. We see that, while in both cases the best agreement is again reached by the POWHEG + PYTHIA combination, the agreement of HEJ is quite good for the measurements where the veto threshold is fixed at 20 GeV, while it is quite bad for larger values of the cut on the veto jet.

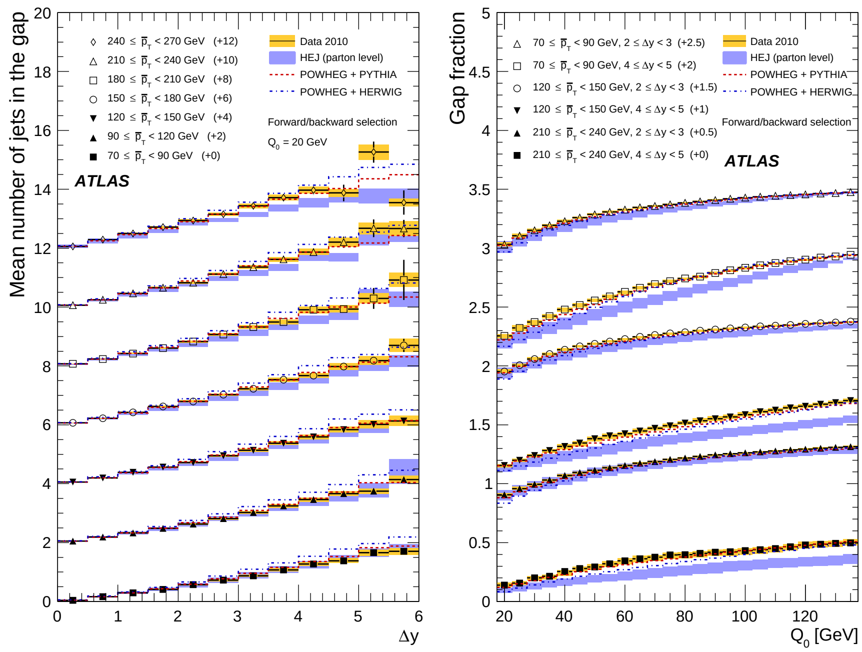

Fig. 6. Gap fraction as a function of the rapidity separation between the most forward and most backward jet in the event. Data is compared with NLO and resummed generators. Left: gap fraction is plotted as a function of the rapidity separation, keeping the threshold of the veto jet at 20 GeV; right: the gap fraction for various combinations of average transverse momentum and rapidity separation of the boundary jets is shown as a function of the threshold of the veto jet.

6. Azimuthal decorrelations for forward dijets in the presence of jet veto

Both azimuthal angle decorrelation and the internal jet veto probe the presence of extra radiation in addition to (and within) the two boundary jets. To make more specific tests of QCD, it makes sense to combine the two requirements for large rapidity separations between the two boundary jets, measuring for instance the azimuthal angle as a function of $\Delta\eta$ for jets with and without extra radiation in the rapidity gap.

CMS produced two results where the azimuthal decorrelation between two jets is measured as a function of their rapidity separation. In Ref.,[30] jets with rapidity separations up to 9.4 are considered. The normalised cross section for jets with transverse momentum above a given threshold p_{Tmin} as a function of the azimuthal angle $\Delta\phi$ can be expanded in a Fourier series:

$$\frac{1}{s}\frac{ds(\Delta y, p_{Tmin})}{d(\Delta\phi)} = \frac{1}{2p}[1 + 2\Sigma_{n=1}^{\inf} C_n(\Delta y, p_{Tmin})\cos(n(p - \Delta\phi))]$$

where the Fourier coefficients C_n are equal to the average of the cosines of the

decorrelation angles multiplied by the order of the series n:

$$C_n(\Delta y, p_{Tmin}) =< \cos(n(p - \Delta\phi)) >$$

At Born level, only two back-to-back jets are present, and at all orders $C_n = 1$. Additional radiation leads to these coefficients becoming smaller than one, and these decorrelations should increase with Δy, due to the wider phase-space available to this extra radiation. The ratios of these average cosines are particularly interesting[31–33] since on the one hand some experimental uncertainties cancel out, on the other some DGLAP contributions are expected to cancel,[32] so BFKL-like effects are expected to be more visible. Figure 7 shows the ratio of the first two cosine coefficients, as a function of the rapidity separation between the most forward and most backward jet in the event. In the left plot data is compared to leading-order generators, and some discrepancy is visible at large values of the rapidity difference, even if the PYTHIA predictions are just on the upper side of the 1-sigma systematic error band. The right plot shows a comparison to a leading-logs BFKL-inspired generator (Cascade 2), to a matrix-element DGLAP generator (SHERPA 1.4), and to a Next-to-Leading Logs (NLL) BFKL analysitcal calculation at parton level.[33] In this specific ratio data is in very good agreement with this calculation, and this would be the first measurement so far showing better agreement with a BFKL-inspired model than a DGLAP one; however the agreement with BFKL NLL+ gets worse in the ratio between coefficients 3 and 2, and in the values of the coefficients themselves, as shown in figures 8.

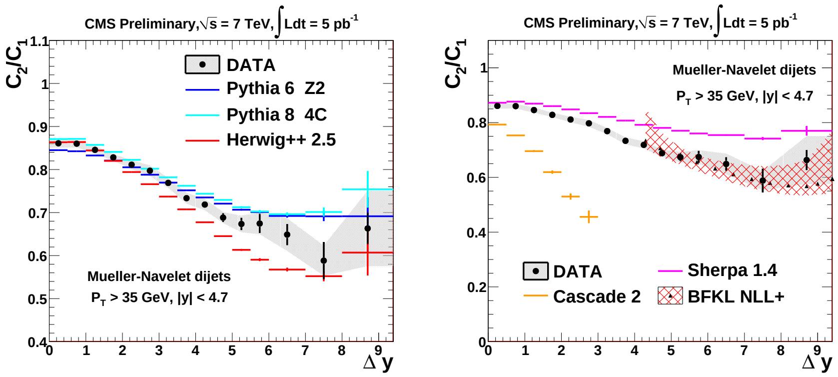

Fig. 7. The ratio of the second and first Fourier coefficients C_2/C_1, as a function of the rapidity separation between the jets. On the left, data is compared to LL DGLAP parton shower generators; on the left, to the multi-leg matrix element generator SHERPA, to the LL BFKL-inspired generator CASCADE, and to a parton-level analytic NLL BFKL calculation.

The second measurement[34] studies the azimuthal de-correlations between a central and a forward jet separately for the cases where another jet is present or for the case of a jet veto. Figure 9 shows the dijet cross section as a function of $\Delta\Phi$ between a central and a forward jet, for various bins of the pseudorapidity separation $\Delta\eta$.

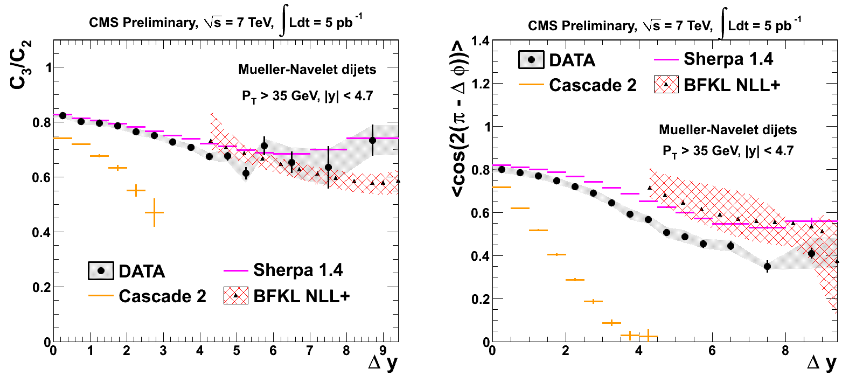

Fig. 8. Left plot: the ratio of the third and second Fourier coefficients C_3/C_2, as a function of the rapidity separation between the jets, and compared to SHERPA, CASCADE, and BFKL NLL+. Right plot: the coefficient C_2, compared to the same theoretical models.

Unfolded data is compared to the LO generators HERWIG 6, HERWIG++ and PYTHIA 8. In general, good agreement is found, within systematic uncertainties that can reach 50% or more, while the difference between the models is about half that value.

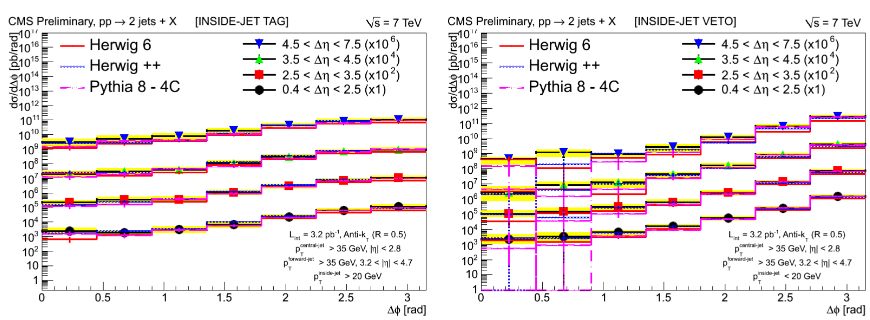

Fig. 9. The dijet cross section as a function of the azimuthal angle between a central and a forward jet in various bins of their pseudorapidity separation $\Delta\eta$. The left plot corresponds to events where an additional jet is present between the two boundary ones, the right one to the jet veto case. Data is compared to LO generators HERWIG 6, HERWIG++ and PYTHIA 8.

Extending the study performed in Ref.[23] to larger ranges in transverse momenta and rapidity separations, ATLAS performed several measurements[27] both for all dijet events and for those with rapidity gaps. Results are compared to the NLO generator POWHEG, where a matrix element calcultion is matched to the parton shower of PYTHIA8 and of HERWIG, as well as to HEJ, at parton level and after showering by ARIADNE. The measurement is performed in various bins of average tranverse momentum of the leading jets, and of their rapidity separation.

Figure 10 shows the ratio of the various theoretical models to data, for vari-

ous bins of rapidity separation between the two leading jets. The left plot is for all events, the right one is for events with a rapidity gap, namely no additional jets above a transverse momentum $Q_0 = 30$ GeV in the rapidity range between the leading jets. As for the CMS measurement, systematic errors (added to the theoretical error bands) can be large; however the difference between the various models can be larger than these uncertainties. In general, apart from the smallest rapidity separation, HEJ tends to underestimate the cross section, but has quite a similar slope with respect to data, while POWHEG, with both showers, albeit being in general in better agreement with data for small $\Delta\Phi$ values, tends as well to underestimate the cross section as the azimuthal angle difference approaches π, so the ratio has a more marked slope.

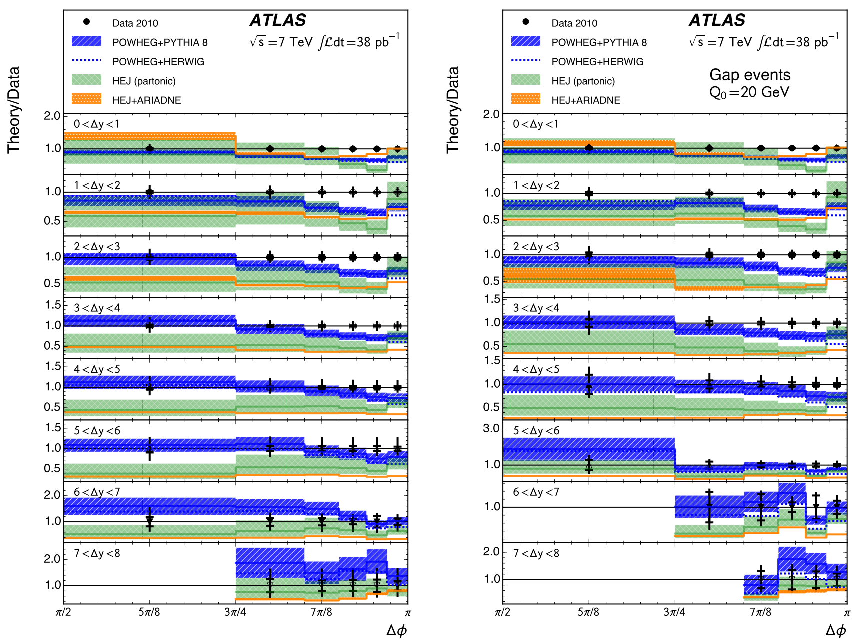

Fig. 10. The dijet cross section as a function of the azimuthal angle difference between the leading dijets in the event. The measurements are presented as ratios between theory predictions and data, for all events (left), and those without a jet with transverse momentum larger than 30 GeV in the rapidity interval between them.

The plots in Fig. 11 show the ratio C_2/C_1 (the same as on Fig. 7, but for a different boundary jet definition) as a function of the rapidity separation between the jets, for all events (left) and for those with a rapidity gap (right). As for the CMS case, the ratio between the Fourier coefficients is found to be a powerful variable to separate the various theoretical models, and a case when data show significant disagreements with the DGLAP approach. In particular the left plot, showing all events, seems to indicate a better agreement with HEJ than with POWHEG, while

this is not so clear in the case (right plot) when a rapidity gap is requested.

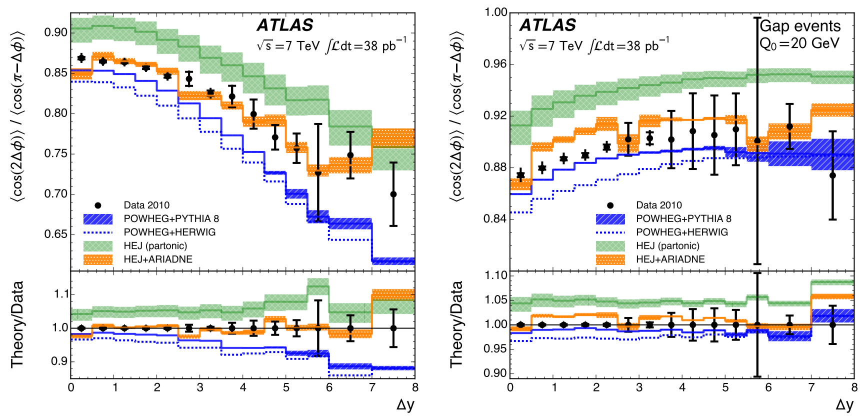

Fig. 11. The ratio of the Fourier coefficients C_2/C_1, as a function of the rapidity separation Δy between the leading dijets in the event. Left plot is for all events, right one is only for the events with a rapidity gap, i.e. without another jet with transverse momentum larger than 30 GeV in the rapidity interval between them.

7. Conclusions

The two general-purpose experiments at the LHC have performed several measurements in particular corners of the jet production phase-space, to test QCD predictions in regions where the simple predictions are expected to badly describe data. Among them, the regime where jets are separated by large rapidity separations, and therefore produced in the forward regions, is expected to be better described by alternative evolution equations, and has been actively investigated. However, in the many measurements presented NLO QCD predictions, formulated within the standard DGLAP framework, still show an amazingly good agreement with data. Only very specific measurements, like the ratios of the coefficients of the Fourier expansion of the difference of the azimuthal angle, start to show discrepancies, and possibly a better agreement with the most recent BFKL-inspired predictions; but further measurements, and better suited observables, are needed to state these conclusions in a more definite way.

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